# **ACT Math Study Guides**

The Complete Guide to ACT Math Book can be overwhelming. With 35 chapters and over 1,250 practice questions, it can be difficult to know where you should focus your studying. Not to worry! We have created this ACT Math Study Guide to help you know what topics to study.

# **How To Use The Study Guides**

There are 4 study guides: one for each of the 4 levels. Each study guide lists the concepts you need to understand and equations you need to memorize based on your math level and scoring goals. If you need to find your level, go back to page iii.

We recommend that you the study guides in 2 ways:

- 1. Use the study guide as a checklist of chapters to complete in the book. As you complete each concept and chapter, you can check them off.
- 2. **Use the study guide to review before test day**. In the 2 weeks before your ACT, go back through the topics you have learned. Make sure that you have all equations memorized, know the concepts you learned, and review any questions you answered incorrectly.

# **Level 1 Study Guide – Core Fundamentals**



Math Level: Easy

Best For: Students Scoring 23 and Below on ACT Math

Prevalence on ACT: Very Common

**Level 1 concepts are the most heavily tested topics on the ACT and the ones that everyone needs to know.** If you are a weaker math student, start by learning all the topics listed below. Level 1 concepts are the easiest to learn and do not require advanced math skills. Questions in core fundamentals most commonly appear in the first 30 questions on the ACT and sometimes appear in questions 30-44 as well. These topics are rarely tested in the most difficult questions at the end of the ACT Math from 45-60.

As a weaker math student, you should focus on trying to answer as many of the first 44 questions (easy and medium difficulty) correctly. These topics will help you do that! You will likely run out of time on the ACT Math Test – that is okay! You should not worry about the more advanced math questions at the end of the test in questions 45-60, as these are designed to stump advanced math students.

# 2 Test-Taking Tricks To Know

**Chapter 1: Backsolving (A Powerful Test-Taking Trick)** 

**Chapter 2: Substitution (Another Test-Taking Trick)** 

# **Almost Guaranteed to Appear on the ACT**

#### **Chapter 3: Geometry Part 1 – Angles**

- o Intersecting Lines (p. 7) memorize rules.
- o Parallel Lines (p. 7) memorize rules, understand example 1.
- O Interior Angles of Polygons (pp. 8-9) memorize equation for total interior angles, understand example 3.
- o Drawn to Scale Trick (p. 9)

# **Chapter 4: Geometry Part 2 – Shapes**

- Area and Volume (p. 14) memorize equations, understand example 1.
- o Areas, Volumes, and Units (pp. 15-16) memorize conversions, understand example 4.
- o Right Triangles (p. 17) memorize Pythagorean Theorem, try to understand example 6.
- o Similar Triangles (p. 19) memorize rules, try to understand example 9.
- o More Triangles (p. 20) memorize definitions of equilateral triangle, isosceles triangle, and scalene triangle, try to understand example 10.

# **Chapter 5: Lines**

- O Slope (p. 31) memorize equation, understand example 1.
- Slopes of Parallel and Perpendicular Lines (pp. 31-32) memorize rules, understand example 3.
- O Slope-Intercept Form (pp. 32-33) memorize equation, understand example 4.
- O Standard Form (p. 34) know how to turn into slope-intercept form, understand example 6.
- o Midpoint Formula (p. 34) memorize equation, understand example 7.
- O Distance Formula (p. 35) memorize equation, understand example 8.
- o Solving for Intercepts (p. 36) memorize rules, understand example 9.

#### **Chapter 6: Fractions**

- O Combining Fractions (p. 41) know how to add or subtract fractions with numbers, do not worry about example 1.
- O Dividing Fractions (pp. 41-42) know the flip and multiply trick, try to understand example 2.
- O Simplifying Fractions (pp. 42-43) understand simplifying with numbers, do not worry if variables examples and example 2 are confusing.
- o Getting Rid of Fractions (p. 44) know how to get rid of fractions if you hate fractions!
- O Turn Fractions Into Decimals (p. 45) great trick to make fractions questions easier.
- O Use Your Calculator (p. 45) know how to turn decimals into fractions with your calculator.

### **Chapter 7: Algebra Skills**

- o PEMDAS (p. 51) understand example 1.
- O Subtracting Negative Numbers (p. 52) understand example 2.
- O Combining Like Terms (p. 53) understand example 4.
- Cross Multiply Fractions (p. 53) understand example 5.
- o Algebra with Inequality Signs (p. 55) memorize rule.

#### **Chapter 8: Number Theory**

- O Types of Numbers (pp. 61-62) memorize definitions, understand example 1.
- o Properties of Even and Odd Numbers (pp. 62-63) understand rules.
- o Greatest Common Factor (p. 63) understand example 4.
- o Least Common Multiple (p. 64) understand example 6.

# **Chapter 9: Percentages**

- o Simple Percentage (p. 72) understand examples 1-2.
- o Percentage Increase and Decrease (pp. 76-77) memorize equations, understand example 1.
- o Percentage Change (p. 77) memorize equation, understand example 4.

# **Chapter 10: Ratios and Proportions**

- o Ratio and a Total (p. 82) memorize the "x" trick", understand example 1.
- o Ratios as Proportions (p. 83) understand example 2.

# **Chapter 11: Functions**

- o Function Basics (p. 89) understand examples 1-2.
- o Composite Functions (pp. 90) understand example 4.
- o Functions on Graphs (p. 92) understand example 8.

# Chapter 12: Mean, Median, Mode, and Range

- o Finding the Average (p. 102) memorize definitions, understand example 1.
- o Finding the Median (p. 105) understand example 7.

# **Chapter 13: Exponents and Roots**

- o Basic Exponent Rules (pp. 112-113) memorize rules, try to understand examples 1-3.
- o Simplifying Square Roots (pp. 114-115) understand example 5.

# **Chapter 16: Quadratics**

- o Multiplying Binomials (p. 135) understand example 1.
- o Factoring Quadratics (p. 136-137) know how the box method works, try to understand example 2.
- o "Easy to Factor" Quadratics (p. 136) memorize equations.
- O Solutions, Roots, x-intercepts, and Zeros for Quadratic Equations (pp. 137-138) understand example 3.

#### **Chapter 17: Trigonometry**

o Basic Trigonometry – SOH-CAH-TOA (pp. 146-147) – understand examples 1-2.

#### **Chapter 19: Matrices**

- O What is a Matrix? (p. 179)
- Matrix Addition and Subtraction (pp. 179-180) understand examples 1-2.

#### **Chapter 22: Probability**

- o The Basics of Probability (pp. 210-211) understand examples 1-2
- o 3 Probability Rules to Know (pp. 211-212) memorize the 3 rules, understand examples 3-5.

# **Chapter 26: Word Problems**

O Understand 4 Tips for Solving Word Problems (p. 250)

# **Sometimes on the ACT (But Easy If You Know The Rules)**

# **Chapter 18: Absolute Value**

O Basics of Absolute Value (p. 173) – understand example 1.

# Chapter 21: Circles, Ellipses, and Hyperbolas

Equation of a Circle (p. 198-199) – memorize equation, understand example 1.

# Chapter 23: Factorial, Permutations, Combinations, and Organized Counting

Organized Counting (p. 226) – understand example 5, do not worry about other examples.

# **Chapter 24: Sequences**

- O Arithmetic Sequences (p. 235) memorize arithmetic sequence definition, understand example 1.
- o Geometric Sequences (p. 236) memorize geometric sequence definition, understand example 3.

# **Chapter 25: Complex Numbers**

- Adding and Subtracting Complex Numbers (p. 241) understand example 1.
- O Use Your Calculator for Complex Numbers (p. 243) see if your calculator has an i button!

# **Chapter 27: Inequalities**

O Algebra with Inequalities (pp. 256-257) – memorize rule, understand examples 1-2.

# **Chapter 29: Unit Conversion**

o Simple Unit Conversions (p. 271) – understand examples 1-2.

#### **Chapter 30: Scientific Notation**

O How To Write Numbers in Scientific Notation (pp. 277-278) – know how to write numbers in scientific notation, understand example 1.

### **Chapter 32: Vectors**

- What is a Vector? (p. 285)
- Vector Addition and Subtraction (p. 286) understand examples 1-2.

# **Level 2 Study Guide – Additional Fundamentals**



Math Level: Easy, Medium

**Best For: Students Scoring 24-27 on ACT Math** 

Prevalence on ACT: Common, Somewhat Common, Less Common

Level 2 concepts range from very commonly tested to less commonly tested and do not require advanced math skills. These are the concepts, rules, and equations you can learn to quickly boost your score. Many of these concepts are ones you may have never learned in school or have forgotten because you only covered them briefly in math class.

This Level 2 study guide includes some Level 1 concepts that students often need to review and additional Level 2 concepts. The Level 2 additional fundamentals in this study guide expand your math knowledge and allow you to answer more questions correctly. Level 2 concepts are of medium difficulty and range from quite commonly tested to less commonly tested, and they are all easy to learn.

Level 2 concepts can appear anywhere in the ACT Math Test, but they most commonly appear in the easy or medium difficulty questions from 11-44. Some topics in this section are "advanced" topics that appear late on the ACT, but they are included in this section because they are not actually too difficult in term of math skills. Instead, these topics are "advanced" simply because they are topics many students are not as familiar with. If you can master these topics, you can answer these questions correctly on test day and boost your math score!!

# 2 Test-Taking Tricks To Know

Chapter 1: Backsolving (A Powerful Test-Taking Trick)

**Chapter 2: Substitution (Another Test-Taking Trick)** 

# Likely To Appear on the ACT

#### **Chapter 3: Geometry Part 1 – Angles**

- o Intersecting Lines (p. 7) memorize rules.
- o Parallel Lines (p. 7) memorize rules, understand example 1.
- O Interior Angles of Polygons (pp. 8-9) memorize equation, understand examples 2-3.
- o Drawn to Scale Trick (p. 9)

#### Chapter 4: Geometry Part 2 – Shapes

- Area and Volume (p. 14) memorize equations, understand examples 1-2.
- o Areas, Volumes, and Units (pp. 15-16) memorize conversions, understand example 4.
- o Right Triangles (p. 17) memorize Pythagorean Theorem, understand example 6.
- o Similar Triangles (p. 19) memorize rules, understand example 9.
- More Triangles (p. 20) memorize definitions of equilateral triangle, isosceles triangle, and scalene triangle, understand example 10.

#### **Chapter 5: Lines**

- O Slope (p. 31) memorize equation, understand examples 1-2.
- O Slopes of Parallel and Perpendicular Lines (pp. 31-32) memorize rules, understand example 3.
- o Standard Form (p. 34) know how to turn into slope-intercept form, understand example 6.
- o Midpoint Formula (p. 34) memorize equation, understand example 7.

# **Chapter 6: Fractions**

- O Combining Fractions (p. 41) know how to add or subtract fractions with numbers, try to understand example 1.
- O Dividing Fractions (pp. 41-42) know the flip and multiply trick, understand example 2.
- O Simplifying Fractions (pp. 42-43) understand simplifying with numbers and variables, understand example 3.
- o Getting Rid of Fractions (p. 44) know how to get rid of fractions if you hate fractions!
- o Turn Fractions Into Decimals (p. 45) great trick to make fractions questions easier.
- Use Your Calculator (p. 45) know how to turn decimals into fractions with your calculator.

# **Chapter 7: Algebra Skills**

- o PEMDAS (p. 51) understand example 1.
- O Subtracting Negative Numbers (p. 52) understand example 2.
- Negative Numbers and Exponents (p. 52) understand how to enter negative numbers and exponents correctly in your calculator.
- o Square Both Sides of an Equation Correctly (p. 54) understand example 6.
- Algebra with Inequality Signs (p. 55) memorize rule, understand example 7.
- Taking Square Roots in Algebraic Equations (p. 56) memorize when to use the ± sign, understand example 8.

#### **Chapter 8: Number Theory**

- O Types of Numbers (pp. 61-62) memorize definitions, understand example 1.
- o Properties of Even and Odd Numbers (pp. 62-63) understand example 3.
- o Greatest Common Factor (p. 63) understand example 4.
- o Least Common Multiple (p. 64) understand example 6.

#### **Chapter 9: Percentages**

- Simple Percentage (p. 72) understand examples 1-2.
- o Percentage Increase and Decrease (pp. 76-77) memorize equations, understand examples 1-2.
- o Percentage Change (p. 77) memorize equation, understand example 4.

#### **Chapter 10: Ratios and Proportions**

- o Ratio and a Total (p. 82) memorize the "x" trick", understand example 1.
- o Ratios as Proportions (p. 83) understand example 2.

#### **Chapter 11: Functions**

- o Function Basics (p. 89) understand examples 1-3.
- o Composite Functions (pp. 90) understand example 4.

#### Chapter 12: Mean, Median, Mode, and Range

- o Finding the Average (pp. 102-103) understand examples 1 and 2.
- o Mean, Median, Mode, and Outliers (p. 105) understand example 6.
- o Finding the Median (p. 105) understand example 7.

### **Chapter 13: Exponents and Roots**

- Basic Exponent Rules (pp. 112-113) memorize rules (especially the fractional exponent rule), understand examples 1-3.
- Simplifying Square Roots (pp. 114-115) understand examples 5-6.

#### **Chapter 14: Logarithms**

- O Basics of Logarithms (pp. 123-124) know how to turn a logarithm to exponential form, understand examples 1-2.
- o Memorize Change of Base Rule (pp. 123-124)

# **Chapter 15: Systems of Equations**

- o Solving with Elimination and Substitution (p. 129) understand examples 1-2.
- O Word Problems (p. 130) understand example 3.

# Chapter 16: Quadratics

- o Multiplying Binomials (p. 135) understand example 1.
- o Factoring Quadratics (p. 136-137) understand example 2.
- o "Easy to Factor" Quadratics (p. 136) memorize equations.
- Solutions, Roots, x-intercepts, and Zeros for Quadratic Equations (pp. 137-138) understand example 3.

# **Chapter 17: Trigonometry**

- O Basic Trigonometry SOH-CAH-TOA (pp. 146-147) understand examples 1-3.
- Using Trigonometry to Find Side Lengths in Right Triangles (pp. 147-148) understand examples 4-5.
- o Inverse Trigonometric Functions (pp. 148-149) understand example 6.
- O Amplitude (p. 151) memorize rule, understand example 8.

# **Chapter 19: Matrices**

- O What is a Matrix? (p. 179)
- o Matrix Addition and Subtraction (pp. 179-180) understand examples 1-2.

### **Chapter 20: Repeating Patterns**

o Repeating Decimals (p. 191) – understand examples 1-2.

#### **Chapter 22: Probability**

- o The Basics of Probability (pp. 210-211) understand examples 1-2.
- o 3 Probability Rules to Know (pp. 211-212) memorize the 3 rules, understand examples 3-5.

### Chapter 23: Factorial, Permutations, Combinations, and Organized Counting

- o Factorial (p. 224) understand example 1 and factorial with numbers.
- o Organized Counting (pp. 226-227) understand examples 5-7.

#### **Chapter 26: Word Problems**

Understand 4 Tips for Solving Word Problems (p. 250)

#### **Chapter 29: Unit Conversion**

o Simple Unit Conversions (p. 271) – understand examples 1-2.

# **Sometimes On The ACT**

# **Chapter 5: Lines**

- o Point-Slope Form (p. 33) memorize equation, understand example 5.
- O Distance Formula (p. 35) memorize equation, understand example 8.
- O Solving for Intercepts (p. 36) memorize rules, understand example 9.

# **Chapter 16: Quadratics**

- O How Solutions Appear on a Graph (pp. 138-139) memorize rules, understand example 5.
- o The Quadratic Formula (p. 139) memorize equation, understand example 6.

# **Chapter 18: Absolute Value**

- o Basics of Absolute Value (p. 173) understand example 1.
- Absolute Value and Unknown Variables (pp. 173-174) understand example 2.

# Chapter 21: Circles, Ellipses, and Hyperbolas

Equation of a Circle (p. 198-199) – memorize equation, understand examples 1-2.

# **Chapter 24: Sequences**

- Arithmetic Sequences (p. 235) memorize arithmetic sequence definition, understand example 1.
- o Geometric Sequences (p. 236) memorize geometric sequence definition, understand example 3.

# **Chapter 25: Complex Numbers**

- o Adding and Subtracting Complex Numbers (p. 241) understand example 1.
- o Multiplying Complex Numbers (pp. 241-242) understand example 2.
- Use Your Calculator for Complex Numbers (p. 243) see if your calculator has an i button!

# **Chapter 30: Scientific Notation**

How To Write Numbers in Scientific Notation (pp. 277-278) – understand examples 1-2.

### **Chapter 32: Vectors**

- O What is a Vector? (p. 285)
- O Vector Addition and Subtraction (p. 286) understand examples 1-2.

# **Less Commonly Tested**

#### **Chapter 4: Geometry Part 2 – Shapes**

Special Right Triangles (p. 18) – memorize triangle proportions, understand example 7.

# **Chapter 8: Number Theory**

- o Prime Factorization (p. 65) understand example 7.
- One Solution vs. Infinite Solutions, vs. No Solution (pp. 66-67) memorize rules, understand example 8.

### **Chapter 10: Ratios and Proportions**

- o Comparing Across Ratios (p. 83) understand example 3.
- o Ratios and Geometry (p. 84) understand example 4.

# **Chapter 11: Functions**

- Other Function Notations (p. 91) understand example 6.
- o Functions on Graphs (pp. 92-93) understand examples 8-9.
- O Domain and Range (pp. 93-94) memorize definitions, understand example 10.

# **Chapter 19: Matrices**

- o Matrix Dimensions (p. 179)
- o Finding the Determinant of a (2 × 2) Matrix (p. 181) memorize equation.

# **Chapter 22: Probability**

- o Probability and Data Tables (p. 213) understand examples 6-7.
- o Expected Value (p. 216) understand example 11.

### **Chapter 27: Inequalities**

- o Algebra with Inequalities (pp. 256-257) memorize rule, understand examples 1-2.
- o Graphing Inequalities (pp. 257-258) memorize rules, understand example 3.

# **Chapter 33: Shifting and Transforming Functions**

o Rules for Shifting and Transforming Functions (p. 291) – memorize rules.

- O Venn Diagrams (pp. 305-306) understand example 1.
- Mapping (p. 308) understand example 5.

# **Rarely Tested**

# **Chapter 10: Ratios and Proportions**

Direct and Indirect Proportions (pp. 84-85) – memorize equations, understand example 5.

# Chapter 12: Mean, Median, Mode, and Range

o Finding the Median in a Table (p. 106) – understand example 8.

# **Chapter 16: Quadratics**

• The Vertex (p. 141) – memorize vertex form equation and how to find x-coordinate of the vertex, understand example 9.

# **Chapter 17: Trigonometry**

o Trigonometric Identity To Know (p. 149) – memorize identity, understand example 7.

# **Chapter 25: Complex Numbers**

o The Complex Conjugate (pp. 242-243) – memorize definition.

# **Chapter 28: Exponential Growth and Decay**

- Exponential Growth and Decay Equations (p. 264) memorize equations.
- o Exponential Growth (pp. 264-265) understand example 1.
- o Exponential Decay (p. 265) understand example 2.
- o General Exponential Form (p. 266) memorize general form and what the constants mean.

### **Chapter 31: Arcs and Sectors**

- Arcs (pp. 280-281) memorize equation, understand examples 1-2.
- o Inscribed Angle Theorem (p. 281) memorize rule.
- o Sectors (pp. 281-282) memorize equation, understand examples 3-4.

#### **Chapter 34: Statistics**

- Description of A Survey (pp. 296-297) memorize definitions and understand example 1.
- O Shapes of Distributions (pp. 297-298) memorize definitions and understand example 2.
- o Stem and Leaf Plot (pp. 299-300) understand example 5.
- o Sample proportion (pp. 300-301) understand example 6.

- o Made-Up Math (p. 306) understand example 2.
- o Logic (p. 307) memorize contrapositive rule, understand example 3.

# **Level 3 Study Guide – Advanced Topics**



Math Level: Medium, Hard

**Best For: Students Scoring 28-32** 

Prevalence on ACT: Common, Somewhat Common, Less Common, Rarely

**Tested** 

If you want to score 30+ on the ACT Math, you need to know how to solve these advanced topics. These advanced topics are the ones that are commonly tested on the difficult and very difficult questions from 45-60. Now, do not let the "advanced" name intimidate you. Yes, some of these topics are very difficult, but many are "advanced" simply because most students do not know them.

If you work through these topics and memorize the equations, you will know how to solve many "advanced" questions at the end of ACT Math Tests from questions 45-60 and will be in the best position to score 30+ on test day!

# 2 Test-Taking Tricks To Know

Chapter 1: Backsolving (A Powerful Test-Taking Trick)

**Chapter 2: Substitution (Another Test-Taking Trick)** 

# **Level 1 and 2 Topics to Make Sure You Know**

Before starting the advanced topics listed on the following pages, make sure you know the topics listed below from the core fundamentals (Level 1) and advanced fundamentals (Level 2). These topics are ones that we have seen many students, even very strong math students, struggle on. Many of these topics are not taught in school or are taught very briefly. To score 30+, you NEED to know all these fundamental topics.

#### **Chapter 3: Geometry Part 1 – Angles**

- O Interior Angles of Polygons (pp. 8-9) memorize equation, understand examples 2-3.
- o Drawn to Scale Trick (p. 9)

### **Chapter 4: Geometry Part 2 – Shapes**

- O Area and Volume (p. 14) memorize equations, understand examples 1-3.
- o Areas, Volumes, and Units (pp. 15-16) memorize conversions, understand example 4.
- o Special Right Triangles (p. 18) memorize triangle proportions, understand example 7.

# **Chapter 5: Lines**

- o Point-Slope Form (p. 33) memorize equation, understand example 5.
- o Midpoint Formula (p. 34) memorize equation, understand example 7.
- O Distance Formula (p. 35) memorize equation, understand example 8.
- o Solving for Intercepts (p. 36) memorize rules, understand example 9.

#### **Chapter 6: Fractions**

- Combining Fractions (p. 41) know how to add or subtract fractions with numbers and variables, understand example 1.
- O Dividing Fractions (pp. 41-42) know the flip and multiply trick, understand example 2.
- Use Your Calculator (p. 45) know how to turn decimals into fractions with your calculator.

# **Chapter 7: Algebra Skills**

- Negative Numbers and Exponents (p. 52) understand how to enter negative numbers and exponents correctly in your calculator, understand example 3.
- o Taking Square Roots in Algebraic Equations (p. 56) − memorize when to use the ± sign, understand example 8.

#### **Chapter 8: Number Theory**

- o Types of Numbers (pp. 61-62) memorize definitions, understand examples 1-2.
- o Greatest Common Factor (p. 63) understand examples 4-5.
- o Least Common Multiple (p. 64) understand example 6.
- One Solution vs. Infinite Solutions, vs. No Solution (pp. 66-67) memorize rules, understand example 8.

### **Chapter 9: Percentages**

- o Simple Percentage (pp. 72-73) understand example 3.
- o Percentage Increase and Decrease (pp. 76-77) memorize equations, understand examples 1-3.
- o Percentage Change (p. 77) memorize equation, understand example 4.

# **Chapter 10: Ratios and Proportions**

- o Ratio and a Total (p. 82) memorize the "x" trick", understand example 1.
- o Comparing Across Ratios (p. 83) understand example 3.
- o Ratios and Geometry (p. 84) understand example 4.
- O Direct and Indirect Proportions (pp. 84-85) memorize the equations, understand examples 5-6.

# **Chapter 11: Functions**

- Composite Functions (pp. 90-91) understand example 5.
- Other Function Notations (p. 91) understand example 6.

#### Chapter 12: Mean, Median, Mode, and Range

- o Finding the Average (pp. 102-104) understand examples 1-5.
- Outliers and Mean, Median, Mode, and Range (p. 105) understand examples 6-7.
- o Finding the Median in a Table (p. 106) understand example 8.

### **Chapter 13: Exponents and Roots**

- O Basic Exponent Rules (pp. 112-113) memorize rules (especially the fractional exponent rule), understand examples 1-3, understand change of base trick for example 4.
- Simplifying Square Roots (pp. 114-115) understand examples 5-6.

#### **Chapter 14: Logarithms**

- O Basics of Logarithms (pp. 123-124) know how to turn a logarithm to exponential form, understand examples 1-2.
- Memorize Change of Base Rule (pp. 123-124)

# **Chapter 15: Systems of Equations**

- o Solving with Elimination and Substitution (p. 129) understand examples 1-2.
- O Word Problems (pp. 130-131) understand examples 3 and 4.

#### **Chapter 16: Quadratics**

- o "Easy to Factor" Quadratics (p. 136) memorize equations, understand example 2.
- o Solutions for Quadratic Equations (pp. 137-138) understand examples 3-4.
- O How Solutions Appear on a Graph (pp. 138-139) memorize rules, understand example 5.
- o The Quadratic Formula (p. 139) memorize equation, understand example 6.

# **Chapter 17: Trigonometry**

- o Basic Trigonometry SOH-CAH-TOA (pp. 146-147) understand examples 1-3.
- Using Trigonometry to Find Side Lengths in Right Triangles (pp. 147-148) understand examples 4 5.
- o Inverse Trigonometric Functions (pp. 148-149) understand example 6.
- o Trigonometric Identity To Know (p. 149) memorize identity, understand example 7.
- o Amplitude (p. 151) memorize rule, understand example 8.

# **Chapter 18: Absolute Value**

Absolute Value and Unknown Variables (pp. 173-174) – understand example 2.

# **Chapter 19: Matrices**

- O What is a Matrix? (p. 179)
- o Matrix Addition and Subtraction (pp. 179-180) understand examples 1-2.
- o Matrix Dimensions (p. 179)
- $\circ$  Finding the Determinant of a  $(2 \times 2)$  Matrix (p. 181) memorize equation, understand example 3.

# **Chapter 20: Repeating Patterns**

o Repeating Decimals (p. 191) – understand examples 1-2.

#### Chapter 21: Circles, Ellipses, and Hyperbolas

o Equation of a Circle (p. 198-199) – memorize equation, understand examples 1-2.

#### **Chapter 22: Probability**

- o 3 Probability Rules to Know (pp. 211-212) memorize the 3 rules, understand examples 3-5.
- o Probability and Data Tables (p. 213) understand examples 6-7.

# **Chapter 23: Permutations, Combinations, and Organized Counting**

- o Factorial (p. 224) understand example 1 and factorial with numbers, do not worry about example 2.
- o Organized Counting (pp. 226-227) understand examples 5-7.

# **Chapter 24: Sequences**

- O Arithmetic Sequences (p. 235) memorize arithmetic sequence definition, understand example 1.
- o Geometric Sequences (p. 236) memorize geometric sequence definition, understand example 3.

# **Chapter 25: Complex Numbers**

- o Multiplying Complex Numbers (pp. 241-242) understand example 2.
- o The Complex Conjugate (pp. 242-243) memorize rule, understand example 3.
- O Use Your Calculator for Complex Numbers (p. 243) see if your calculator has an i button!

#### **Chapter 26: Word Problems**

o Understand 4 Tips for Solving Word Problems (p. 250)

# **Chapter 27: Inequalities**

o Graphing Inequalities (pp. 257-258) – memorize rules, understand example 3.

### **Chapter 28: Exponential Growth and Decay**

- o Exponential Growth and Decay Equations (p. 264) memorize equations.
- o Exponential Growth (pp. 264-265) understand example 1.
- o Exponential Decay (p. 265) understand example 2.
- o General Exponential Form (p. 266) memorize general form and what the constants mean.

#### **Chapter 30: Scientific Notation**

o How To Write Numbers in Scientific Notation (pp. 277-278) – understand examples 1-2.

# **Chapter 31: Arcs and Sectors**

- o Arcs (pp. 280-281) memorize equation, understand examples 1-2.
- o Inscribed Angle Theorem (p. 281) memorize rule.
- o Sectors (pp. 281-282) memorize equation, understand examples 3-4.

# **Chapter 32: Vectors**

- What is a Vector? (p. 285)
- O Vector Addition and Subtraction (p. 286) understand examples 1-2.

# **Chapter 34: Statistics**

- O Description of A Survey (pp. 296-297) memorize definitions and understand example 1.
- O Shapes of Distributions (pp. 297-298) memorize definitions and understand example 2.
- O Stem and Leaf Plot (pp. 299-300) understand example 5.
- o Sample proportion (pp. 300-301) understand example 6.

- Venn Diagrams (pp. 305-306) understand example 1.
- o Made-Up Math (p. 306)
- o Logic (p. 307) memorize contrapositive rule.
- o Mapping (p. 308) understand example 5.

# **Advanced Topics for Level 3 Students**

The advanced topics listed below are the ones that are commonly tested on the difficult and very difficult questions from 45-60 at the end of the ACT Math Test. We recommend that you work down the list in the order it is presented: start with the topics that you are likely to see on test day and work your way to the rarely tested topics. If you have limited time preparing for the ACT, do not worry about completing the "Rarely Tested" topics.

Mastering the advanced topics and memorizing the equations listed below will prepare you for the advanced questions at the end of ACT Math Tests and put you in the best position to score 30+ on test day!

# Likely to See on Test Day

# **Chapter 17: Trigonometry**

- o Law of Sines (pp. 158-159) memorize equation, understand example 12.
- o Law of Cosines (pp. 160) memorize equation, understand example 14.

# **Chapter 19: Matrices**

o Matrix Multiplication (pp. 182-186) – understand examples 4-5.

# **Chapter 21: Circles and Ellipses**

- o Advanced Circle Questions (pp. 199-200) memorize 2 principles, understand examples 3-4.
- o Ellipses (pp. 201-202) memorize ellipse equation, know how to draw an ellipse given the equation, understand examples 5-6.
- o Foci on an Ellipse (p. 203) know where the foci are located.

# **Chapter 22: Probability**

o Advanced Probability Questions (pp. 214-215) – understand examples 8-10.

# **Sometimes on the ACT**

# **Chapter 4: Geometry Part 2 – Shapes**

- o Volume of Other Three-Dimensional Solids (p. 16) memorize equation, understand example 5.
- o Third Side of a Triangle Rule (p. 21) memorize rule, understand example 11.
- Other Rules You Might Need To Know (p. 22) memorize all rules and equations.

#### Chapter 12: Median, Mode, and Range

Weighted Average (p. 104) – memorize equation, understand examples 3-4.

#### **Chapter 13: Exponents and Roots**

- O Cube Roots (pp. 115-116) understand example 7.
- o Roots and Variables with Powers (p. 116) understand example 8.

#### **Chapter 29: Unit Conversion**

o Dimensional Analysis (pp. 272-273) – understand examples 3-4.

# **Chapter 32: Vectors**

- Vector Addition and Subtraction on A Graph (pp. 286-287) know "tip-to-tail" method, understand example 3.
- o Magnitude of a Vector (p. 288) memorize definition and equation.

# **Less Commonly Tested**

# **Chapter 17: Trigonometry**

- o Graphing Cosine and Sine Functions (p. 150) know basics about each graph.
- o Period (pp. 152-153) memorize rule, understand example 9.
- O Vertical and Horizontal Shifts for Sine and Cosine Functions (pp. 154-155)
- o Summary of Transformation Rules for Sine and Cosine Functions (p. 155) memorize rules.

# **Chapter 20: Repeating Patterns**

- o Repeating Patterns with Powers and the Units Digit (pp. 192-193) understand example 3, recognize commonly tested repeating patterns with powers and the units digit (do not need to memorize)
- $\circ$  Repeating Patterns with Powers of i (p. 194) memorize pattern, understand example 5.
- o How to Solve Repeating Patterns Questions (p. 195)

# Chapter 23: Factorial, Permutations, Combinations, and Organized Counting

- o Permutations (p. 225) know how to use your calculator to solve, understand example 3.
- o Combinations (p. 226) know how to use your calculator to solve, understand example 4.

# **Chapter 27: Inequalities**

o Advanced Graphing Inequalities (pp. 259-260) – memorize rules for shading, understand example 4.

#### **Chapter 33: Shifting and Transforming Functions**

- o Rules for Shifting and Transforming Functions (p. 291) memorize rules.
- o Graphing and Shifting Cubics (p. 292)
- o Shifting All Other Functions (p. 293)

# **Rarely Tested**

# **Chapter 11: Functions**

- O Domain and Range (pp. 93-95) memorize rules, understand examples 10-13.
- o Inverse Functions (p. 96) understand example 14.

### **Chapter 14: Logarithms**

- o Logarithm Rules (p. 124-125) memorize rules, understand example 3.
- o Natural Logarithms (p. 125) understand example 5.

#### **Chapter 16: Quadratics**

- o The Discriminant (p. 140) memorize rules, understand example 7.
- The Vertex (pp. 141-142) memorize vertex form equation and how to find x-coordinate of the vertex, understand examples 9-10.

### **Chapter 17: Trigonometry**

Unit Circle (pp. 161-163) – refamiliarize yourself with the unit circle. You do not need to memorize
the entire thing. Memorize 3 basic principles of the unit circle and the "All Students Take Calculus"
rule. Understand example 16.

### **Chapter 18: Absolute Value**

O Absolute Value and Inequalities (pp. 174-175) – understand examples 3-4.

# **Chapter 22: Probability**

o Expected Value (pp. 216-217) – memorize equation, understand examples 11-12.

#### **Chapter 24: Sequences**

Recursive Sequences (p. 237) – understand how to use recursive sequences and example 4.

# **Chapter 25: Complex Numbers**

The Complex Plane (p. 244) – know how to graph complex numbers on the complex plane, understand example 4.

# **Chapter 34: Statistics**

Standard Deviation (pp. 298-299) – memorize rule, understand examples 3-4

- o Pattern Spotting (p. 307-308) understand example 4.
- o Binomial Theorem and Pascal's Triangle (pp. 310-311) understand example 7. You do not need to know how to draw Pascal's Triangle.
- Vertical Asymptotes (p. 313) memorize rule, understand example 10.

# **Level 4 Study Guide – Expert Level Topics**



Math Level: Hard, Expert

Best For: Students Aiming For 33-36 on ACT Math Prevalence on ACT: Less Common, Rarely Tested

If your goal is to score 33-36 on the ACT Math, you need to know EVERYTHING tested on the ACT. That means you should understand EVERYTHING in this book. While knowing everything is a challenging task given how many topics the ACT Math includes, Level 4 concepts cover the most common obscure and rarely tested topics that you can see on test day and the additional equations you should memorize. All these topics have appeared on the ACT before, so they are fair game to appear on your ACT.

Before you work on Level 4 concepts, makes sure that you know all the topics in the Level 3 study guide and have all the equations memorized. All topics in this section are very advanced and rarely tested, so you should only move onto these topics once you have mastered everything in the Level 3 study guide.

# **Important Tips to Achieve 33-36**

If you are aiming for 33-36, make sure you know how to solve all level 4 questions at the end of each chapter. These questions are written specifically for students like you and will help you learn how to solve the most advanced questions that appear on the ACT.

The difficult questions at the end of each ACT Math Test (questions 45-60) can include any topic. Most commonly, it is a mix of advanced topics (Ex: matrix multiplication, ellipses, law of sines, etc.) and regular topics (ratios, averages, quadratics, geometry, etc.) presented in a difficult way. Therefore, it is important to make sure you work through the level 2-4 question in all chapters of this book even if the general topic of the chapter seems easy.

To achieve a 33-36, you need to (1) have all the equations and rules memorized for test day and (2) be familiar with all topics on the ACT so you can set up and solve questions quickly and efficiently on test day. Time management can be the difference between a 32 and a 35. Putting in the time and working through all chapters of this book will put you in the best position for achieving your goals on test day!

# 2 Test-Taking Tricks To Know

**Chapter 1: Backsolving (A Powerful Test-Taking Trick)** 

**Chapter 2: Substitution (Another Test-Taking Trick)** 

# **Expert Level Topics to Know**

Before starting these topics, go to the Level 3 Study Guide and make sure you know all Level 3 topics! There are some level 2 and level 3 topics included in the list below. These are topics that we see many strong math students struggle with because the topics are not commonly taught in school or are the ones students commonly forget. All topics listed below are very important ones to master for anyone aiming for 33-36 on the ACT.

# **Chapter 4: Geometry Part 2 – Shapes**

- O Volume of Other Three-Dimensional Solids (p. 16) memorize equation, understand example 5.
- Special Right Triangles (p. 18) understand example 8.
- o Third Side of a Triangle Rule (p. 21) memorize rule, understand example 11.
- Other Rules You Might Need To Know (p. 22) memorize all rules and equations

#### **Chapter 7: Algebra Skills**

- Negative Numbers and Exponents (p. 52) understand how to enter negative numbers and exponents correctly in your calculator, understand example 3.
- O Taking Square Roots in Algebraic Equations (p. 56) memorize when to use the ± sign, understand example 8.

# **Chapter 8: Number Theory**

- o Types of Numbers (pp. 61-62) memorize definitions, understand examples 1-2.
- One Solution vs. Infinite Solutions, vs. No Solution (pp. 66-67) memorize rules.

# **Chapter 10: Ratios and Proportions**

- o Comparing Across Ratios (p. 83) understand example 3.
- O Direct and Indirect Proportions (pp. 84-85) memorize the equations, understand examples 5-6.

#### **Chapter 11: Functions**

- Other Function Notation (pp. 91-92) understand example 7.
- O Domain and Range (pp. 93-95) memorize rules and understand examples 11-13.
- o Inverse Functions (p. 96) understand example 14.

#### Chapter 12: Median, Mode, and Range

- o Weighted Average (pp. 103-104) memorize equation, understand examples 3-4.
- o Finding the Average (pp. 104-105) understand example 5.

# **Chapter 13: Exponents and Roots**

- O Basic Exponent Rules (pp. 112-113) memorize rules (especially the fractional exponent rule), understand change of base trick for example 4.
- O Cube Roots (pp. 115-116) understand example 7.
- o Roots and Variables with Powers (p. 116) understand example 8.

#### **Chapter 14: Logarithms**

- o Memorize Change of Base Rule (pp. 123-124)
- o Logarithm Rules (pp. 124-125) memorize rules, understand examples 3-4.
- O Natural Logarithms (p. 125) understand example 5.

# **Chapter 15: Systems of Equations**

O Word Problems (pp. 130-131) – understand examples 3 and 4.

# **Chapter 16: Quadratics**

- o The Quadratic Formula (p. 139) memorize equation, understand example 6.
- o The Discriminant (p. 140) memorize rules, understand examples 7-8.
- The Vertex (pp. 141-142) memorize vertex form equation and how to find x-coordinate of the vertex, understand examples 9-10.

# **Chapter 17: Trigonometry**

- o Trigonometric Identity To Know (p. 149) memorize identity, understand example 7.
- o Graphing Cosine and Sine Functions (p. 150) know basics about each graph.
- o Period (pp. 152-153) memorize rules, understand example 9.
- O Vertical and Horizontal Shifts for Sine and Cosine Functions (pp. 154-155) understand example 10.
- o Summary of Transformation Rules for Sine and Cosine Functions (p. 155) memorize rules.
- o Graphing Tangent Functions (pp. 156-157) know basics of graph and period rules, understand example 11.
- o Law of Sines (pp. 158-159) memorize equation, understand examples 12-13.
- o Law of Cosines (pp. 160) memorize equation, understand example 14.
- Unit Circle (pp. 161-163) memorize the unit circle, memorize 3 basic principles of the unit circles and the "All Students Take Calculus" rule, understand examples 15-16.

# **Chapter 18: Absolute Value**

Absolute Value and Inequalities (pp. 174-175) – know how to set up and solve, understand examples
 3-4.

# **Chapter 19: Matrices**

- o Matrix Dimensions (p. 179)
- $\circ$  Finding the Determinant of a  $(2 \times 2)$  Matrix (p. 181) memorize equation, understand example 3.
- o Matrix Multiplication (pp. 182-186) understand examples 4-5.

#### **Chapter 20: Repeating Patterns**

- o Repeating Decimals (p. 191) understand examples 1-2.
- Repeating Patterns with Powers and the Units Digit (pp. 192-193) understand examples 3-4, recognize commonly tested repeating patterns with powers and the units digit (do not need to memorize)
- o Repeating Patterns with Powers of i (p. 194) memorize pattern, understand examples 5-6.
- How to Solve Repeating Patterns Questions (p. 195)

### Chapter 21: Circles, Ellipses, and Hyperbolas

- o Equation of a Circle (p. 198-199) memorize equation, understand examples 1-2.
- o Ellipses (pp. 201-202) memorize ellipse equation, know how to draw an ellipse given the equation, understand examples 5-6.
- o Foci on an Ellipse (p. 203) understand examples 6-7.
- Hyperbolas (pp. 204-205) memorize equations for vertical and horizontal hyperbolas, understand example 8.

#### **Chapter 22: Probability**

- o 3 Probability Rules to Know (pp. 211-212) memorize the 3 rules, understand examples 3-5.
- o Advanced Probability Questions (pp. 214-215) understand examples 8-10
- o Expected Value (pp. 216-217) memorize equation, understand examples 11-12

# **Chapter 23: Permutations, Combinations, and Organized Counting**

- o Factorial (p. 224) understand factorial definition and examples 1-2.
- o Permutations (p. 225) know how to use your calculator to solve, understand example 3.
- o Combinations (p. 226) know how to use your calculator to solve, understand example 4.
- o Organized Counting (p. 227) understand examples 6-7.
- Advanced Combinations, Permutations, and Organized Counting (pp. 228-229) understand examples 8-9.

#### **Chapter 24: Sequences**

- o Arithmetic Sequences (p. 235) memorize equation and understand example 2.
- o Geometric Sequences (p. 236) memorize equation at bottom of page.
- o Recursive Sequences (p. 237) understand how to use recursive sequences and example 4.
- o Explicit Formula for Recursive Sequences (p. 238) understand example 5.

# **Chapter 25: Complex Numbers**

- o The Complex Conjugate (pp. 242-243) memorize rule, understand example 3.
- O Use Your Calculator for Complex Numbers (p. 243) see if your calculator has an i button!
- The Complex Plane (p. 244) know how to graph complex numbers on the complex plane, understand example 4.
- o Absolute Value of a Complex Number (pp. 244-245) memorize equation, understand example 5.
- Distance Between Complex Numbers (p. 245) memorize equation, understand example 6.

#### **Chapter 26: Word Problems**

O Understand 4 Tips for Solving Word Problems (p. 250)

#### **Chapter 27: Inequalities**

 Advanced Graphing Inequalities (pp. 259-260) – memorize rules for shading, understand examples 4-5.

#### **Chapter 28: Exponential Growth and Decay**

- o Exponential Growth and Decay Equations (p. 264) memorize equations.
- o Exponential Growth (pp. 264-265) understand example 1.
- o Exponential Decay (p. 265) understand example 2.
- o General Exponential Form (p. 266) memorize general form and what the constants mean.
- o Graphing General Exponential Form (pp. 266-267)
- Advanced Exponential Growth and Decay Equations (p. 267) memorize equations, understand example 3.

### **Chapter 29: Unit Conversion**

O Dimensional Analysis (pp. 272-273) – understand examples 3-5.

# **Chapter 31: Arcs and Sectors**

- o Arcs (pp. 280-281) memorize equation, understand examples 1-2.
- o Inscribed Angle Theorem (p. 281) memorize rule.
- o Sectors (pp. 281-282) memorize equation, understand examples 3-4.

### **Chapter 32: Vectors**

- O What is a Vector? (p. 285)
- Vector Addition and Subtraction (p. 286) understand examples 1-2.
- Vector Addition and Subtraction on A Graph (pp. 286-287) know "tip-to-tail" method, understand example 3.
- o Magnitude of a Vector (p. 288) memorize definition and equation.

# **Chapter 33: Shifting and Transforming Functions**

- o Rules for Shifting and Transforming Functions (p. 291) memorize rules.
- o Graphing and Shifting Cubics (p. 292)
- o Shifting All Other Functions (p. 293)

### **Chapter 34: Statistics**

- o Description of A Survey (pp. 296-297) memorize definitions and understand example 1.
- O Shapes of Distributions (pp. 297-298) memorize definitions and understand example 2.
- Standard Deviation (pp. 298-299) memorize rule, understand examples 3-4.
- o Stem and Leaf Plot (pp. 299-300) understand example 5.
- o Sample proportion (pp. 300-301) understand example 6.
- O Normal Distribution and The Empirical Rule (p. 301) memorize rules, understand example 7.

- o Logic (p. 307) memorize contrapositive rule, understand example 3.
- o Pattern Spotting (p. 307-308) understand example 4.
- o Puzzles (p. 309) understand example 6.
- o Binomial Theorem and Pascal's Triangle (pp. 310-311) know how to draw Pascal's Triangle, understand examples 7-8.
- O Visual Spatial (p. 312) understand how to solve example 9.
- Asymptotes (pp. 313-315) memorize vertical and horizontal asymptote rules, understand examples 10-11.